Control of Temporal Constraints Based on Dioid Algebra for Timed Event Graphs

S. Amari, J.J. Loiseau

IRCCyN, UMR CNRS 6597
Ecole Centrale de Nantes
loiseau@irccyn.ec-nantes.fr

and I. Demongodin

LISA, FRE CNRS 2656
Université d'Angers
isabel.demongodin@univ-angers.fr

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Outline

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- Min-Plus linear models of timed event graphs, their properties.
- Strict temporal constraints in Min-Plus
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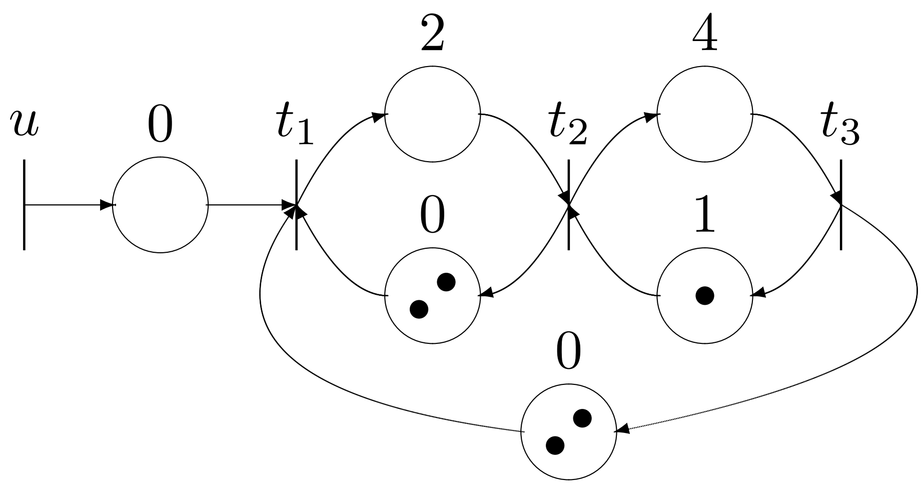
Introduction

The main features of this work are the following.

- The problem is to respect strict temporal constraints.
- We use a simple linear model over the Min-Plus semiring.
- The design of a control ensuring the constraint validation is proposed.
- It is actually an on-line admission control.
- The work originated from a real industrial manufacturing workshop.

Timed event Graphs



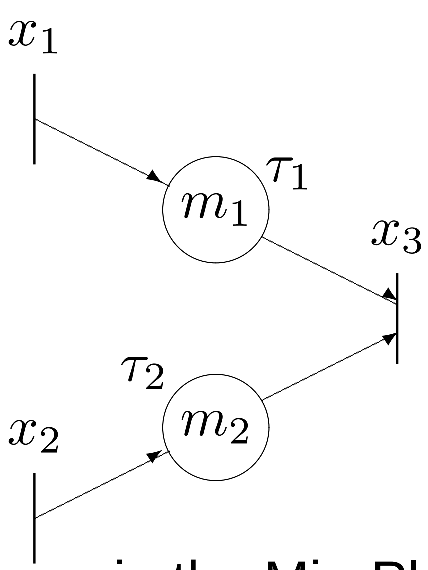
A P-timed event graph

- Petri Net = Places $\in \mathcal{P}$ + Transitions $\in \mathcal{T}$ + Arcs $\in \mathcal{P} \times \mathcal{T} \cup \mathcal{T} \times \mathcal{P}$, + Initial tokens, + marking evolution rules.
- Event Graph = Each place has exactly one downstream and one upstream transition.
- *Timed* Event Graph = A delay, denoted τ_{ij}, is associated to the place from t_j to t_i, if any. It is a minimal sojourn time for the tokens.

Min-Plus linear model : Principle

- To each transition is associated a counter
  = $u_i(t)$ for a source transition, $\theta_i(t)$ for the other transitions.

For instance, the timed event graph



leads to the equation
$\theta_3(t) = \min(m_1 + \theta_1(t - \tau_1), m_2 + \theta_2(t - \tau_2))$,

that reads
$\theta_3(t) = m_1\theta_1(t - \tau_1) \oplus m_2\theta_2(t - \tau_2)$,

in the Min-Plus algebra $\overline{\mathbb{R}}_{\min} = (\mathbb{R} \cap \{\infty\}, \min = \oplus, +)$.

Min-Plus linear model

In general, we obtain a behavioral model of the form

$$\theta(t) = \bigoplus_{\tau=0}^{\tau^{\max}} \left(A_\tau \cdot \theta(t-\tau) \oplus B_\tau \cdot u(t-\tau) \right) ,$$

where $\tau^{\max}$ is the maximal delay occuring in the graph. The model can be rewrited as

$$\theta(t) = \bigoplus_{\tau>0} \left(A_0^\star \cdot A_\tau \cdot \theta(t-\tau) \oplus A_0^\star \cdot B_\tau \cdot u(t-\tau) \right) ,$$

where $A_0^\star := \bigoplus_{k\in\mathbb{N}} A_0^k$ is the Kleene star of A_0.

See : F. Baccelli, G. Cohen, G.-J. Olsder and J.-P. Quadrat, Synchronization and linearity, Wiley, 1992.

Min-Plus linear model (cont.)

Some hypotheses are done.

- (H_1) All the delays equal 0 or 1.
  This leads to a state-space representation: $x(t) = Ax(t-1) \oplus Bu(t)$,
  $\theta(t) = Cx(t);$.
- (H_2) The input $u(t)$ is a control. One can postpone the firing of the source transitions, then it is an admission control.
- (H_3) There is only one input, for the sake of simplicity.

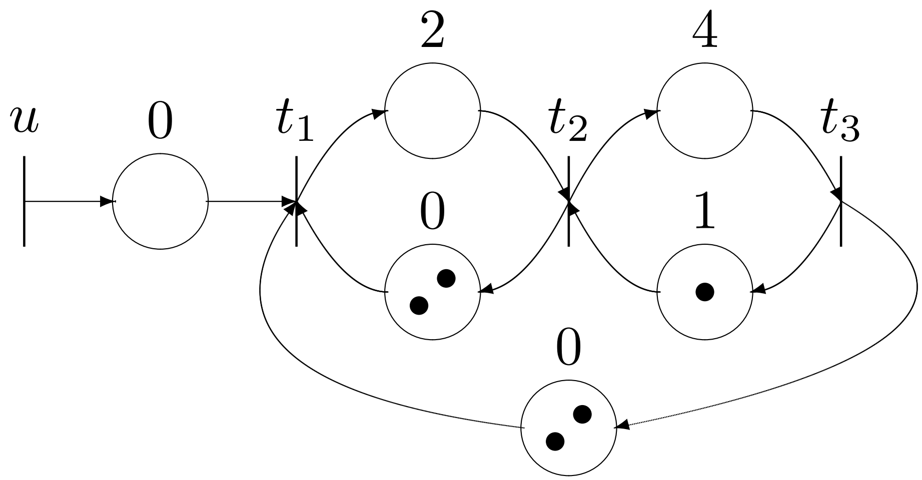
The main consequence is that the system is causal, deterministic, linear, and in state-space form. Hence

$$x(t) = A^{\tau} \cdot x(t-\tau) \oplus \left[\bigoplus_{k=0}^{\tau-1} A^k \cdot B \cdot u(t-k) \right] ,$$

holds true, for every integer $\tau \geq 1$.

Example

Consider again the previous example,



a P-timed event graph.

For this graph, the basic Min-Plus linear equation reads

$$\begin{aligned}\theta(t) &= A_0\theta(t) \oplus A_1\theta(t-1) \\ &\oplus A_2\theta(t-2) \oplus A_4\theta(t-4) \oplus Bu(t)\ ,\end{aligned}$$

Example (cont.)

with

$$A_0 = \begin{pmatrix} \epsilon & 2 & 2 \\ \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \end{pmatrix}, A_1 = \begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & 1 \\ \epsilon & \epsilon & \epsilon \end{pmatrix},$$

$$A_2 = \begin{pmatrix} \epsilon & \epsilon & \epsilon \\ e & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \end{pmatrix}, A_4 = \begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \\ \epsilon & e & \epsilon \end{pmatrix},$$

and

$$B = \begin{pmatrix} e \\ \epsilon \\ \epsilon \end{pmatrix}.$$

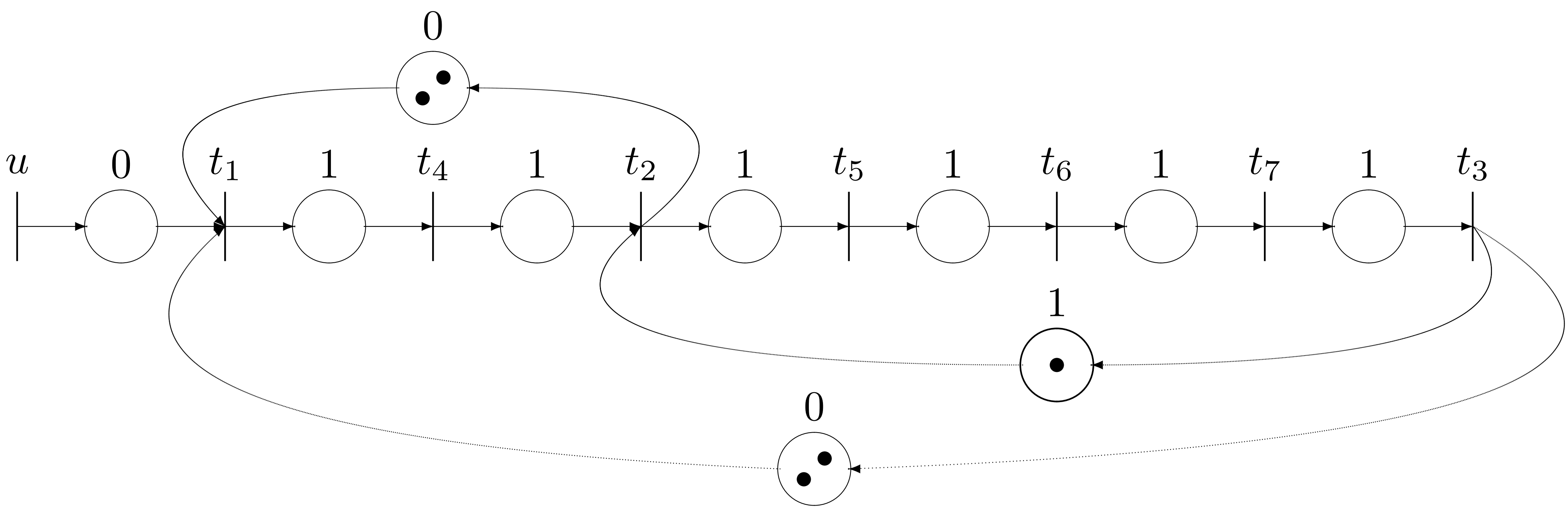
Example (cont.)

Multiplying by $A_0^\star$ the other matrices, one obtains the following explicit equation

$$\theta(t) = \begin{pmatrix} \epsilon & \epsilon & 3 \\ \epsilon & \epsilon & 1 \\ \epsilon & \epsilon & \epsilon \end{pmatrix} \theta(t-1) \oplus \begin{pmatrix} 2 & \epsilon & \epsilon \\ e & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \end{pmatrix} \theta(t-2)$$
$$\oplus \begin{pmatrix} \epsilon & 2 & \epsilon \\ \epsilon & \epsilon & \epsilon \\ \epsilon & e & \epsilon \end{pmatrix} \theta(t-4) \oplus \begin{pmatrix} e \\ \epsilon \\ \epsilon \end{pmatrix} u(t) \ .$$

Example (cont.)

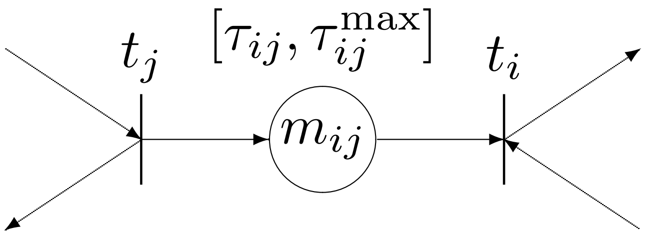
Extending the initial graph to get a graph with delays normalized to 0 or 1, one obtains the following graph



and the resulting state equation is

$$x(t) = \begin{pmatrix} \epsilon & \epsilon & 3 & 2 & \epsilon & \epsilon & 2 \\ \epsilon & \epsilon & 1 & e & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & e \\ e & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & e & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & e & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & e & \epsilon \end{pmatrix} x(t-1) \oplus \begin{pmatrix} e \\ \epsilon \\ \epsilon \\ \epsilon \\ \epsilon \\ \epsilon \\ \epsilon \end{pmatrix} u(t).$$

Taking into account strict temporal constraints



Let p_{ij} be subject to a temporal constraint.

The constraint is expressed through the following inequality:

$$m_{ij} + x_j(t - \tau_{ij}) \geq x_i(t) \geq m_{ij} + x_j(t - \tau_{ij}^{\max}) \ ,$$

where m_{ij} is the initial marking of the place p_{ij}. Since the left inequality is already taken into account by the linear model, the right one, say

$$x_i(t) \geq m_{ij} x_j(t - \tau_{ij}^{\max}) \ ,$$

where the product is over $\overline{\mathbb{R}}_{\min}$, actually represents the constraint.

Causal feedback

We want to calculate $F \in \overline{\mathbb{R}}_{\min}^{m \times N}$ such that

$$u(t) = F \cdot x(t-1) ,$$

for $t > 1$, where the product is in the sense of the Min–Plus algebra, ensures the respect of the constraint

$$x_i(t) \geq m_{ij} x_j(t - \tau_{ij}^{\max}) .$$

(H$_4$) There exists a path α from t_u to t_j.
The corresponding delay is denoted τ_α.

Control synthesis

Combining

$$x_j(t) \leq A_{ju}^{\tau_\alpha} u(t-\tau_\alpha)$$

and

$$x_i(t) = \bigoplus_{r=1}^{N} A_{ir}^{\phi} x_r(t-\phi) \oplus \left[\bigoplus_{k=0}^{\phi-1} (A^k B)_i u(t-k)\right] ,$$

one obtains the main result.

Theorem 1 The constraint $x_i(t) \geq m_{ij} x_j(t-\tau_{ij}^{\max})$ is satisfied taking

$$u(t) \leq \bigoplus_{r=1}^{N} (A_{ir}^{\phi} - A_{ju}^{\tau_\alpha} - m_{ij} x_r(t-1)) ,$$

where $\phi = \tau_{ij}^{\max} + \tau_\alpha + 1$, if the two following conditions hold

(i) $A_{ir}^{\phi} \geq A_{ju}^{\tau_\alpha} + m_{ij}$, for $r = 1$ to N,

(ii) $(A^k B)_i \geq A_{ju}^{\tau_\alpha} + m_{ij}$, for $k = 0$ to $\phi - 1$.

Control synthesis (cont.)

Corollary If there is neither initial token along the path from t_u to t_j, nor in p_{ij}, then the two conditions hold, and

$$u(t) = \bigoplus_{r=1}^{N} A_{ir}^{\phi} x_r(t-1)$$

is a control law that validates the temporal constraint

$$x_i(t) \geq m_{ij} x_j(t - \tau_{ij}^{\max}) \ .$$

Notice that a degree of freedom exists, since the path is not unique, in general.

Generalization

to the case of several temporal constraints, say

$$x_z^{'}(t) \geq m_z x_z(t - \tau_z^{\max}) \ ,$$

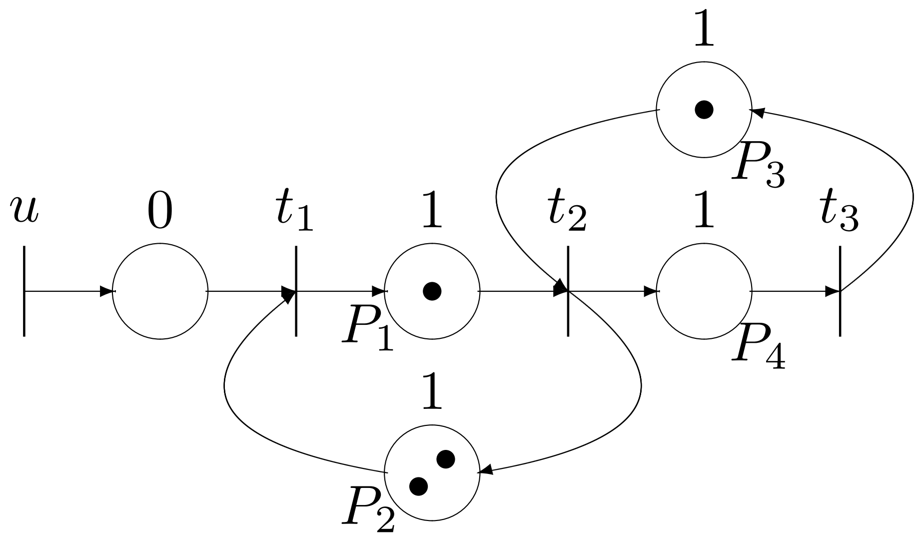
for $z = 1$ to Z.

Theorem 2 The control law

$$u(t) = \bigoplus_{z=1}^{Z} u_z(t) \ ,$$

where $u_z(t)$ is calculated from Theorem 1 for each constraint, defines a causal control that ensures the respect of the Z different temporal constraints.

Example



A furnace and a robot

The state equation associated with this little manufacturing system is

$$x(t) = \begin{pmatrix} \epsilon & 2 & \epsilon \\ 1 & \epsilon & 1 \\ \epsilon & e & \epsilon \end{pmatrix} x(t-1) \oplus \begin{pmatrix} e \\ \epsilon \\ \epsilon \end{pmatrix} u(t) \ ,$$

where the components of $x(t)$ are the counter functions associated to the transitions t_1, t_2, and t_3, and $u(t)$ is the control.

Example (cont.)

Further, the time constraint is expressed in terms of an inequality, say

$$x_2(t) \geq 1 \cdot x_1(t-1) \ ,$$

for $t \geq 1$. Then applying Theorem 1, we obtain $\tau_{ij}^{max} = \tau_{21}^{\max} = 1$, $\tau_\alpha = 0$, $m_{ij} = m_{21} = 1$, and $\phi = \tau_{21}^{\max} + \lambda + 1 := 2$.

We can then check that
(i) $A_{1u}^e + m_{21} = 1$, and $A_{2r}^2 = \epsilon, 1, \epsilon$ respectively, for $r = 1, 2, 3$,
(ii) $(AB)_2 = 1$. The two conditions of the theorem hold, and

$$u(t) = \bigoplus_{r=1}^{3} \left(A_{2,r}^2 - 1\right) x_r(t-1) := x_2(t-1)$$

guarantees that the time constraint is respected.

Conclusions

- A novel approach to the validation of temporal constraints is proposed.
- It consists of a online admission control that ensures the respect of the constraint.
- The computations are easy, thanks to the use of the Min-Plus linear model. *See http://www.istia.univ-angers.fr/ ∼hardouin/ for a software that permits effective computation on large examples*.
- A generalization to the case of several constraints was proposed.

- Other generalizations motivate present studies, to the case of several inputs, or to the case of noncommensurable delays.
  See the forthcoming PhD report of Said Amari.